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## Stationary vibrations and fatigue failure of compressor disks of variable thickness

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**Abstract.** The purpose of the research is calculating the stress-strain state of the elastic compressor disks of gas turbine engine due to vibrations of blades. The thickness of the disks is variable along the radius. The solution presented by Fourier series and the Fourier coefficients are found from the boundary value problems for systems of ordinary differential equations along the radial coordinate. The obtained results are used to estimate the durability of the disks in very-high-cycle fatigue mode.

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*Key words:* disk of variable thickness, stress-strain state, high frequency loading, very-high-cycle fatigue, durability estimation

### Nomenclature

$r, \vartheta, z$  - polar coordinates  
 $h(r)$  - variable thickness of disk  
 $\sigma_{ij}$  - components of stress  
 $\rho$  - density  
 $u_r, u_\vartheta, u_z$  - components of displacement  
 $\lambda, \mu$  - Lamé elastic moduli  
 $\varepsilon_{ij}$  - components of strain  
 $\omega$  - vibration frequency  
 $a$  - internal radius of disk  
 $b$  - external radius of disk  
 $d$  - section width  
 $\gamma$  - intensity of torsion  
 $N_\theta$  - number of blades on disk  
 $\sigma_{\max}$  - maximum stress in cycle  
 $\sigma_{\min}$  - minimum stress in cycle  
 $\beta$  - index of fatigue durability  
 $\sigma_{\text{mean}}$  - mean stress in cycle  
 $\Delta\tau$  - range of shear stress per cycle  
 $\sigma_B$  - ultimate tensile strength

$\sigma_u$  - fatigue limit

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## 1. Introduction

The compressor disks of a gas turbine are permanently loaded by the oscillating blades and this may lead to failure in the very-high-cycle fatigue mode (VHCF) Shanyavskiy (2007). The frequency of these oscillations is of the order of frequency of rotation of the disc, or a multiple of it. Development of VHCF with the number of cycles to failure  $N > 10^8$  may cause appearance of failure in the vicinity of the contact area of the blades and the outer rim of the disc.

Earlier in Burago (2011) for low-cycle fatigue regime (LCF) the stress-strain state and the fatigue durability estimation were calculated for a rotating disk of variable thickness under the action of centrifugal loads in the disk and blades and aerodynamic loads of air flow on blades. Such loads correspond to flight cycles (takeoff-flight-landing). Aerodynamic pressures were calculated using the hypothesis of "isolated profile". Based on the calculated stress state and criteria of multiaxial fatigue fracture of titanium alloys the locations and the dates of origin of fatigue damage were identified.

In calculations it was found that the fatigue life of the titanium disk for typical rotation frequencies in the vicinity of the contact area of the blades and the disk rim may be drastically reduced to the critical values of  $N \sim 10^4$  flight cycles (30000-50000 real time hours). However, along with the LCF mode (flight cycles) there are also acting the low-amplitude cyclic loads of VHCF mode due to blade vibrations. Acting for a long time, the vibrations can also cause the destruction of structures. It is important to note that in the long service life the values of the stresses caused by vibrations are substantially lower than yield limit and fatigue limit. According to the classic views such loadings generally should not pose a danger. But the fractographic study of fracture surfaces destroyed during operation discs showed that the initial nucleation of fatigue microcracks may occur in both modes of LCF and VHCF Shanyavskiy (2007). A distinctive feature of VHCF mode is that the center of damage zone is situated under the surface of the structural element but is not adjacent to the surface as in the case of LCF mode. These features allow experimenters to distinguish between these mechanisms in the classification of the primary damage reason. Note that after appearance of damage zone the further development of a fatigue crack occurs rapidly and is observed in the form of grooves on the surface of fracture. It follows that mentioned modes of fatigue failure are alternative and often complementary and mutually reinforcing each other. A review of experimental research in this direction can be found in Shanyavskiy (2007), Bathias (2005).

The purpose of the given work consists in an estimation of durability of a disk of variable thickness in VHCF mode (vibrations) together with LCF mode. We shall emphasize that at long-term cyclic loadings the structure works within the limits of elasticity and plastic effects are not observed down to the beginning of damage. Therefore for calculation of durability by criteria of fatigue fracture it is enough to solve an elasticity theory problem and to compute a range of stress and strain values in cyclic process.

It is necessary to notice, that in LCF mode the loadings of a disk is provided by centrifugal forces in disk and blades, which also are loaded by aerodynamic pressure of a gas stream. On this stress field the stresses due to vibrations of blades are imposed in VHCF mode. In the accepted statement of problem we do not consider the reason of vibrations, such as action of pulsations of pressure, excitation of own forms of blades vibration, and others to that similar, instead we consider vibrations set as predefined taking the data on amplitudes and frequencies of vibrations observed in practice and presented in Shanyavskiy (2007).

By virtue of linearity of an elasticity theory problem the stress state of flight cycles and the stress state of vibrations, it is possible to calculate separately and then by summation to receive the full stress state. Full stress state from flight cycles and vibrations for two extreme positions of blade at vibrating torsion are the borders of studied cyclic process and these values are used in criteria of fatigue durability. On the basis of multiaxial fatigue criterion the time to fracture and the zones of origin of fatigue failure are found.

There are no experimentally proved, standard criteria for time to failure detection in VHCF mode. Therefore for estimations of durability of VHCF fracture ( $N > 10^8$ ) the generalization Burago (2016) of known multiaxial LCF failure criterion Sines (1959) is used.

## 2. The approximate system of the equations for a disk of variable thickness under action of periodic loadings on external disk boundary.

Let in cylindrical system of coordinates  $r, \vartheta, z$  the ring disk  $a \leq r \leq b$  has variable thickness  $2h(r)$ . The coordinate along thickness varies in limits  $-h(r) \leq z \leq h(r)$ . The system of equations of dynamics theory of elasticity in cylindrical coordinates has the following view:

$$\frac{\partial \sigma_{rr}}{\partial r} + \frac{1}{r} \frac{\partial \sigma_{r\theta}}{\partial \theta} + \frac{\partial \sigma_{rz}}{\partial z} + \frac{\sigma_{rr} - \sigma_{\theta\theta}}{r} = \rho \frac{\partial^2 u_r}{\partial t^2}$$

$$\frac{\partial \sigma_{r\theta}}{\partial r} + \frac{1}{r} \frac{\partial \sigma_{\theta\theta}}{\partial \theta} + \frac{\partial \sigma_{\theta z}}{\partial z} + \frac{2\sigma_{r\theta}}{r} = \rho \frac{\partial^2 u_\theta}{\partial t^2}$$

$$\frac{\partial \sigma_{rz}}{\partial r} + \frac{1}{r} \frac{\partial \sigma_{\theta z}}{\partial \theta} + \frac{\partial \sigma_{zz}}{\partial z} + \frac{\sigma_{rz}}{r} = \rho \frac{\partial^2 u_z}{\partial t^2}$$

Stresses and strains are subjected to the Hooke's law:

$$\begin{aligned} \sigma_{rr} &= (\lambda + 2\mu)\varepsilon_{rr} + \lambda\varepsilon_{\theta\theta} + \lambda\varepsilon_{zz} & \sigma_{\theta\theta} &= \lambda\varepsilon_{rr} + (\lambda + 2\mu)\varepsilon_{\theta\theta} + \lambda\varepsilon_{zz} & \sigma_{r\theta} &= 2\mu\varepsilon_{r\theta} \\ \sigma_{zz} &= \lambda\varepsilon_{rr} + \lambda\varepsilon_{\theta\theta} + (\lambda + 2\mu)\varepsilon_{zz} & \sigma_{rz} &= 2\mu\varepsilon_{rz} & \sigma_{\theta z} &= 2\mu\varepsilon_{\theta z} \end{aligned}$$

Relations between strains and displacements are:

$$\begin{aligned} \varepsilon_{rr} &= \frac{\partial u_r}{\partial r} & \varepsilon_{\theta\theta} &= \frac{1}{r} \frac{\partial u_\theta}{\partial \theta} + \frac{u_r}{r} & \varepsilon_{r\theta} &= \frac{1}{2} \left( \frac{1}{r} \frac{\partial u_r}{\partial \theta} + \frac{\partial u_\theta}{\partial r} - \frac{u_\theta}{r} \right) \\ \varepsilon_{zz} &= \frac{\partial u_z}{\partial z} & \varepsilon_{rz} &= \frac{1}{2} \left( \frac{\partial u_r}{\partial z} + \frac{\partial u_z}{\partial r} \right) & \varepsilon_{\theta z} &= \frac{1}{2} \left( \frac{1}{r} \frac{\partial u_z}{\partial \theta} + \frac{\partial u_\theta}{\partial z} \right) \end{aligned}$$

where  $\lambda$ ,  $\mu$  are Lamé elastic moduli,  $\rho$  is density of disk material. Further the dimensionless stresses are divided by  $\lambda + 2\mu$  while the dimensionless spatial variables and displacements are divided by internal radius of a disk  $a$ . The boundary conditions on free boundaries at  $z = \pm h(r)$  are:

$$\sigma_{rz} - h' \sigma_{rr} = 0 \quad \sigma_{\theta z} - h' \sigma_{r\theta} = 0 \quad \sigma_{zz} - h' \sigma_{rz} = 0$$

Internal boundary of disk is unloaded:

$$r = a: \quad \sigma_{rr} = 0, \quad \sigma_{r\theta} = 0, \quad \sigma_{rz} = 0$$

On the external disk boundary the periodic (along the angular coordinate  $\theta$  and time  $t$ ) applied loads  $\sigma_{r\theta\_b}$  and  $\sigma_{rz\_b}$  are produced by torsional vibrations of the blades:

$$r = b: \quad \sigma_{rr} = 0, \quad \sigma_{r\theta} = \sigma_{r\theta\_b}, \quad \sigma_{rz} = \sigma_{rz\_b}$$

Because of the periodicity of unknown functions along the angular coordinate  $\theta$  and time  $t$  the displacements are represented in the form of Fourier series:

$$u_r = e^{i\omega t} \sum_{n=1}^{\infty} (u_n z + u_{3n} z^3) \sin n\theta \quad u_\theta = e^{i\omega t} \sum_{n=0}^{\infty} (v_n z + v_{3n} z^3) \cos n\theta \quad u_z = e^{i\omega t} \sum_{n=1}^{\infty} (w_n + w_{2n} z^2 + w_{4n} z^4) \sin n\theta$$

Appropriate representations of the stresses are:

$$\begin{aligned} \sigma_{rr} &= e^{i\omega t} \sum_{n=1}^{\infty} (\sigma_n z + \sigma_{3n} z^3) \sin n\vartheta & \sigma_{\vartheta\vartheta} &= e^{i\omega t} \sum_{n=1}^{\infty} (s_n z + s_{3n} z^3) \sin n\vartheta & \sigma_{zz} &= e^{i\omega t} \sum_{n=1}^{\infty} (\Sigma_n z + \Sigma_{3n} z^3) \sin n\vartheta \\ \sigma_{r\vartheta} &= e^{i\omega t} \sum_{n=0}^{\infty} (\tau_n z + \tau_{3n} z^3) \cos n\vartheta & \sigma_{rz} &= e^{i\omega t} \sum_{n=1}^{\infty} (p_n + p_{2n} z^2 + p_{4n} z^4) \sin n\vartheta & \sigma_{\vartheta z} &= e^{i\omega t} \sum_{n=0}^{\infty} (T_n + T_{2n} z^2 + T_{4n} z^4) \cos n\vartheta \end{aligned}$$

The coefficients of the Fourier series are new unknown functions of the radial variable  $r$ , multiplier  $e^{i\omega t}$  defines vibrations,  $\omega$  is a vibration frequency. If substitute the expressions for displacements and stresses in the original system and equate terms of like powers  $z$  up to  $z^3$  then one can obtain the resolution system of ordinary differential equations for the auxiliary variables at different  $n$ .

$$\begin{aligned} \sigma_n' &= n\tau_n / r - (\sigma_n - s_n) / r - 2p_{2n} - \rho\omega^2 u_n & \tau_n' &= -ns_n / r - 2\tau_n / r - 2T_{2n} - \rho\omega^2 v_n \\ p_n' &= nT_n / r - p_n / r - \Sigma_n - \rho\omega^2 w_n & u_n' &= \sigma_n - \lambda U_n / r - \lambda w_{2n} \\ v_n' &= \tau_n / \mu + V_n / r & w_n' &= p_n / \mu - u_n \\ w_{2n}' &= p_{2n} / \mu - 3u_{3n} & p_{2n}' &= nT_{2n} / r - p_{2n} / r - 3\Sigma_{3n} - \rho\omega^2 w_{2n} \\ s_n &= \lambda u_n' + U_n / r + 2\lambda w_{2n} & \Sigma_n &= \lambda u_n' + \lambda U_n / r + 2w_{2n} & T_n &= \mu v_n + \mu n w_n / r \\ u_{3n}' &= \sigma_{3n} - \lambda U_{3n} / r - 4\lambda w_{4n} & \sigma_{3n}' &= n\tau_{3n} / r - (\sigma_{3n} - s_{3n}) / r - p_{4n} - \rho\omega^2 u_{3n} \\ v_{3n}' &= \tau_{3n} / \mu + V_{3n} / r & \tau_{3n}' &= -ns_{3n} / r - 2\tau_{3n} / r - 4T_{4n} - \rho\omega^2 v_{3n} \\ S_{3n} &= \lambda u_{3n}' + U_{3n} / r + 4\lambda w_{4n} & \Sigma_{3n} &= h'^2 (\sigma_n h + \sigma_{3n} h^3) / h^3 - \Sigma_n / h^2 & T_{2n} &= \mu n w_{2n} / r + 3\mu v_{3n} \\ T_{4n} &= h' (\tau_n h + \tau_{3n} h^3) / h^4 - (T_n + T_{2n} h^2) / h^4 & p_{4n} &= h' (\sigma_n h + \sigma_{3n} h^3) / h^4 - (p_n + p_{2n} h^2) / h^4 \end{aligned}$$

where  $U_n = -nv_n + u_n$ ,  $V_n = -nu_n + v_n$ ,  $U_{3n} = -nv_{3n} + u_{3n}$ ,  $V_{3n} = -nu_{3n} + v_{3n}$ .

### 3. The boundary conditions for torsional vibrations

For calculations of stress state for disk due to torsional vibrations of blades the boundary conditions for auxiliary variables (Fourier coefficients) on radial boundaries  $r = a$  and  $r = b$  are

$$\begin{aligned} r = a: & \quad \sigma_n = 0, \quad \sigma_{3n} = 0, \quad \tau_n = 0, \quad \tau_{3n} = 0, \quad p_n = 0, \quad p_{2n} = 0 \\ r = b: & \quad \sigma_n = 0, \quad \sigma_{3n} = 0, \quad \tau_n = \tau_{bn}, \quad \tau_{3n} = 0, \quad p_n = p_{bn}, \quad p_{2n} = -p_{bn} / h^2 \end{aligned}$$

where  $\tau_{bn}$  and  $p_{bn}$  are predefined values of Fourier coefficients for disk boundary stresses in root sections of blades undergoing to torsion. For calculation of values  $\tau_{bn}$  and  $p_{bn}$  consider every blade as a plate of rectangular section of width  $d$ , intensity of torsion  $\gamma$  and use the approximate solution of known task about torsion of plates of rectangular cross section:

$$\tau_b(\vartheta) = Q_0 (1 - \vartheta^2 / \delta^2), \quad Q_0 = -0.8\mu\gamma d / h, \quad |\vartheta| \leq \delta$$

$$p_b(\vartheta) = T_0 \vartheta / \delta, \quad T_0 = \mu\gamma d, \quad |\vartheta| \leq \delta$$

where  $\delta = d / (2b) \ll 1$ .

Let the number of blades on the disk is equal to  $N_0$ . Expand the periodic distribution function of the tangential stress  $\sigma_{r\theta_b}$  and axial shear stress  $\sigma_{rz_b}$  on external boundary (for  $r = b$ ) in Fourier series (one period  $-\pi / N_0 < \vartheta < \pi / N_0$ )

$$\tau_b(\vartheta) = \sum_{k=0}^{\infty} \tau^{(k)} \cos(kN_0\vartheta), \quad \tau^{(0)} = 2Q_0N_0\delta / (3\pi), \quad \tau^{(k)} = \frac{4Q_0}{\pi k^2 N_0 \delta} \left( \frac{\sin(kN_0\delta)}{kN_0\delta} - \cos(kN_0\delta) \right)$$

$$p_b(\vartheta) = \sum_{k=1}^{\infty} p^{(k)} \sin(kN_0\vartheta), \quad p^{(k)} = \frac{2}{k\pi} T_0 \left( \frac{\sin(kN_0\delta)}{kN_0\delta} - \cos(kN_0\delta) \right)$$

where  $n=0, N_0, 2N_0, 3N_0$

Thus, for various  $n$  there must be solved two-point boundary value problems for systems of ordinary differential equations with boundary conditions. The solution of these boundary value problems are determined by numerical method of orthogonal successive substitution. After that, the stresses are determined by summing Fourier series. The number of members in the summation of Fourier series for practical convergence does not exceed 20.

**4. Examples of calculation of disk stress state due to vibrations**

The half-section of disk at  $\vartheta = const$  is shown in Fig. 1. Accepted the following values of geometry and material parameters for disk and blades:  $a = 0.05m, b = 0.4m, d = 0.01m, h=0.035m, \gamma=0.1 \text{ rad/m}, \omega = 628 \text{ 1/s}, \lambda = 78 \text{ GPa}, \mu = 44 \text{ GPa}, \rho = 4370 \text{ kg/m}^3$  (titanium alloy). Number of blades is  $N_n=32$ .

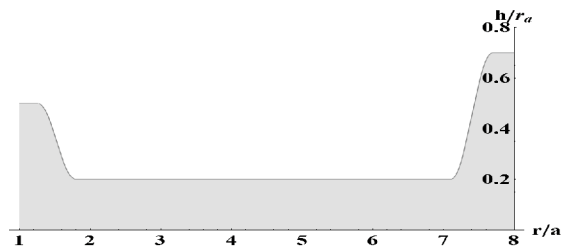


Fig. 1. Shape of disk section at  $\vartheta = const$

The obtained values of the stress amplitudes in torsional vibrations of the blades with the signs + and - were superimposed on the basic stress state associated with the flight cycles of disk loading taking into account the additional aerodynamic loads calculated in Burago (2011).

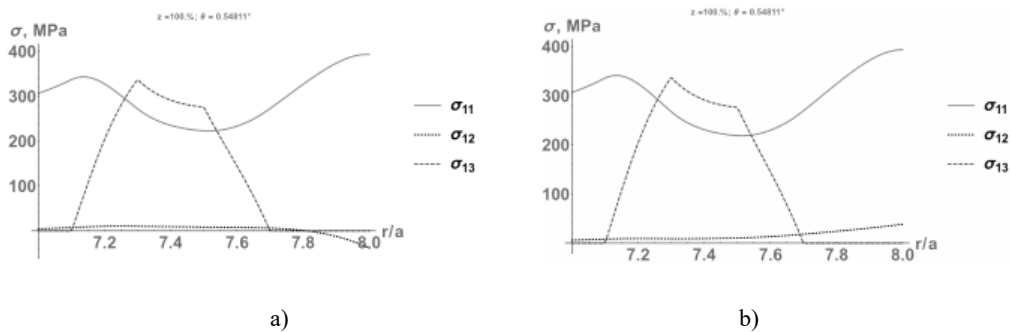


Fig. 2. The total radial distribution of stresses  $\sigma_{11}, \sigma_{12}, \sigma_{13}$

Radial distributions of the six stress components for two extreme positions (graphs (a) and (b)) of torsional cycles of blades vibration in the vicinity of the outer rim of the disc are shown in Fig. 2 and Fig. 3. In the notation of stress the indices 1,2,3 correspond to the coordinates  $r, \theta, z$ .

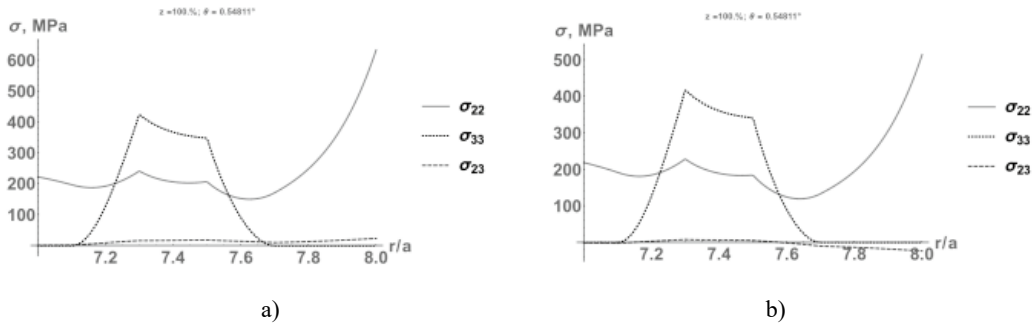


Fig. 3. The total radial distribution of stresses  $\sigma_{22}, \sigma_{33}, \sigma_{23}$ .

The difference between the values of stresses on the left (a) and right (b) graphs in these figures represents the scope of stresses in high-frequency cycle associated with the torsional vibrations of blades. Further, these data are used in the criteria of fatigue fracture in order to find the zones of origin of the damage and to estimate life duration of structures.

**5. Zones of failure origin and an estimation of service life duration in VHCF mode**

There are some criteria and models of multiaxial fatigue fracture in a LCF mode allowing to estimate number of loading cycles to damage of the material sample or an element of a structure Burago (2011). As a basis for definition of parameters for models of multiaxial fatigue fracture the experimental uniaxial cyclic curves are used. These tests are conducted at various values of cycle asymmetry parameter  $R = \sigma_{min} / \sigma_{max}$ , where  $\sigma_{max}$  and  $\sigma_{min}$  are the maximal and minimal values of a stress in a cycle. At the description of results of uniaxial tests for fatigue durability following designations are accepted:  $\sigma_a = (\sigma_{max} - \sigma_{min}) / 2$  is a stress amplitude in a cycle,  $\Delta\sigma = \sigma_{max} - \sigma_{min}$  is a range of stress in a cycle. Experimental data of uniaxial tests are described by Wohler's curves which can be analytically presented by Baskin's relations Burago (2011)  $\sigma = \sigma_u + \sigma_c N^\beta$ , where  $\sigma_u$  is a fatigue limit,  $\sigma_c$  is a fatigue durability coefficient,  $\beta$  is an index of fatigue durability,  $N$  is a number of cycles to damage. The typical kind of a fatigue curve corresponds to the left branch (at  $N < 10^7$ ) of bimodal curve presented in a Fig. 4.

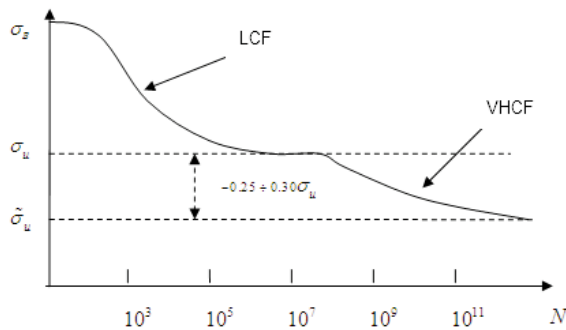


Fig. 4. Bimodal fatigue curve for LCF and VHCF modes

The research problem of fatigue failure consists in definition of spatial distribution of function of number of cycles before destruction  $N$  using generalized for multiaxial state Baskin's type relations and calculated strains and stresses in the structure under consideration. One of the standard generalizations of a uniaxial fatigue curve in LCF mode to multiaxial state is the Sines' criterion Sines (1959):

$$\Delta\tau / 2 + \alpha_s \sigma_{mean} = S_0 + AN^\beta, \quad \sigma_{mean} = (\sigma_1 + \sigma_2 + \sigma_3)_{mean},$$

$$\Delta\tau = \sqrt{(\Delta\sigma_1 - \Delta\sigma_2)^2 + (\Delta\sigma_1 - \Delta\sigma_3)^2 + (\Delta\sigma_2 - \Delta\sigma_3)^2} / 3$$

where  $\sigma_{mean}$  is a sum of principal stresses  $\sigma_1, \sigma_2, \sigma_3$  averaged per loading cycle,  $\Delta\tau$  is a change of shear stress per cycle;  $\Delta\tau/2$  is its amplitude;  $\alpha_s, S_0, A, \beta$  are experimentally defined parameters.

Model parameters calculated by using uniaxial fatigue curves at  $R = -1$  and  $R = 0$  are presented in Burago (2011):

$$S_0 = \sqrt{2}\sigma_u / 3, A = 10^{-3\beta}\sqrt{2}(\sigma_B - \sigma_u) / 3, \alpha_s = \sqrt{2}(2k_{-1} - 1) / 3, k_{-1} = \sigma_u / (2\sigma_{u0})$$

where  $\sigma_u$  and  $\sigma_{u0}$  are fatigue limits according to curves  $\sigma_a(N)$  at  $R = -1$  and  $R = 0$  respectively,  $\sigma_B$  is the strength limit.

The right branch of bimodal fatigue curve (Fig. 4) at  $N > 10^8$  corresponds to VHCF regime (very high cycle fatigue) Shanyavskiy (2007), Bathias (2005). We see that when the number of cycles of the order  $10^9 \div 10^{10}$  the fatigue failure of an element of structure may happen at stress level much less than classical LCF limit Burago (2011) which corresponds to flat area between left and right branches of Wohler’s curves. Experimentally proved multiaxial criteria of VHCF mode are still absent. Therefore, to estimate the durability we use the known Sines’ criterion generalized for VHCF mode and assumption of similarity between the left and right branches of bimodal fatigue curves, proposed in Burago (2016).

In order to account the similarity between the left and right branches of bimodal fatigue curves let’s introduce substitutions  $\sigma_B \rightarrow \sigma_u, \sigma_u \rightarrow \tilde{\sigma}_u, \sigma_{u0} \rightarrow \tilde{\sigma}_{u0}$ , where  $\tilde{\sigma}_u$  and  $\tilde{\sigma}_{u0}$  are «new» fatigue limits for the right branch of fatigue curve at asymmetry coefficients  $R = -1$  and  $R = 0$ . As a result the generalized model parameters for VHCF mode may be written as Burago (2016):

$$S_0 = \sqrt{2}\tilde{\sigma}_u / 3, A = 10^{-8\beta}\sqrt{2}(\sigma_u - \tilde{\sigma}_u) / 3, \alpha_s = \sqrt{2}(2k_{-1} - 1) / 3, k_{-1} = \tilde{\sigma}_u / (2\tilde{\sigma}_{u0})$$

In VHCF calculations the following values of model parameters are used (titanium alloy):  $\sigma_u = 350\text{MPa}, \tilde{\sigma}_u = 250\text{MPa}, \tilde{\sigma}_{u0} = 200\text{MPa}, \beta = -0.3$ .

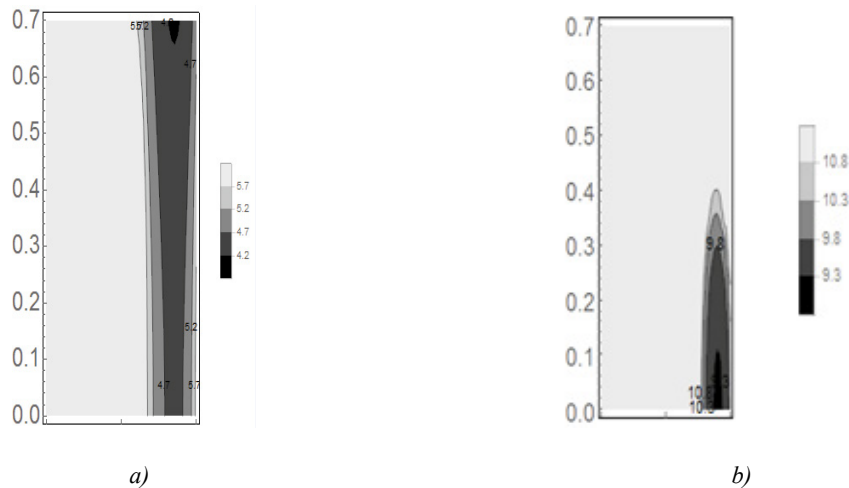


Fig. 5. Isolines of logarithm of durability in rectangular root cross-section of contact zone at outer rim of disk under a blade (at  $r=b$ )

Fig. 5 shows the distribution of logarithm of durability in rectangular contact zone between outer rim of the disc and a blade at  $r = b$  (root cross-section of blade) for total LCF (a) and VHCF (b) modes. Dark color selects regions with minimal durability which corresponds to zones of failure origin. For three hour flights the LCF durability  $N=10^{4.2}$  is estimated as 40000-50000 real time hours (Fig. 5-a). Fig. 5-b shows a significant (up to  $10^{9.3} - 10^{10}$  cycles) drop of durability in the zone of contact between disc and blades. The vibrations have a period of about 0.01s. Therefore, the actual time to fatigue failure due to vibrations of the blades may reach a value of 20 000 - 30 000 hours which are quite achievable during operation. Thus, the both LCF and VHCF mechanisms are alternative and they may cause the fatigue fracture of the element of structure for close service life durations.

## Conclusions

A method for calculating three-dimensional stress-strain state of elastic disks of variable thickness is developed. The cyclic loads due to the torsional vibrations of the blades of the compressor disks and cyclic loads due to flight loading cycles (take-off, flight and landing) are taken into account.

The power series approximate representation is used for spatial dependence of solution along the thickness of the disk and Fourier series are used for representation in the angular direction. The coefficients of such representation formulas depend on the radial coordinate and subjected to the system of ordinary differential equations. The appropriate boundary value problems are solved.

Calculated stress state from vibrations is imposed on to strain-stress state from flight cycles. The resulting total stress state is used for determination of service life of structure under consideration on the basis of generalized Sine's fatigue criterion. The significant drop in the durability on the outer rim of disk in the zone of contact between disc and blades are found.

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